Comparison Between a Generic Reaction-Diffusion Model and a Synergetic Semiconductor System

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Z. Naturforsch. **42 a**, 655–656 (1987);
received March 21, 1987

This paper gives a concrete physical interpretation of a simple dynamical model based on the universal Rashevsky-Turing theory of symmetry-breaking morphogenesis in terms of spatio-temporal nonlinear transport phenomena in a synergetic semiconductor system.

Based on the well-established Rashevsky-Turing (RT) theory of morphogenesis [1], a simple two-compartment chemical reaction-diffusion model has been demonstrated to reflect general behavioral characteristics of many different synergetic systems in nature [2–6]. In the case of a real physical system, we have presented experimental evidence that spatio-temporal nonlinear transport phenomena in semiconductors can be discussed in terms of the basic self-organizing cooperative processes occurring in a spatially distributed RT model, namely symmetry-breaking phase transitions and boiling-type turbulence [5]. In the following, we give a first concrete physical interpretation of the RT prototype model reaction-diffusion system.

As the essential behavioral characteristic of RT systems, breakdown of symmetry can in the simplest case be realized by a two-cellular symmetrical morphogenetic system consisting of cross-inhibitorily coupled, potentially oscillating two-variable subsystems (4-D flow) [3]. The reaction scheme of such a simple RT system is sketched in Figure 1. The two morphogens A are self-inhibiting via B and, to a lesser extent, cross-inhibiting each other via the symmetrical coupling between the two morphogens B. The excess of self-inhibition within each cell over the cross-inhibition generated by the other cell is compensated for, within either cell, by a path of self-activation (autocatalysis of A) which is not mediated through diffusion to the other side. The effects of constant pools and reaction partners are comprised in the effective rate constants $K_1, \ldots, K_5$.

Under the assumption of a Michaelis-Menten-type kinetics the system of Fig. 1 can be described by the following set of simultaneous ordinary differential

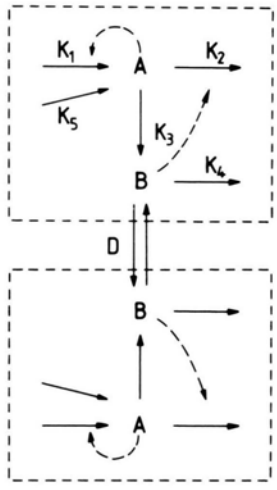

Fig. 1. Reaction scheme of a simple Rashevsky-Turing model system. Constant pools (source and sinks) are omitted from the scheme. Catalytic rate control is indicated by dashed arrows.

equations:

$$\begin{aligned}
\dot{a} &= (K_1 - K_3)\, a - K_2\, b\, \frac{a}{a+K} + K_5\,, \\
\dot{b} &= K_3\, a - K_4\, b + D\,(b' - b)\,, \\
\dot{b}' &= K_3\, a' - K_4\, b' + D\,(b - b')\,, \\
\dot{a}' &= (K_1 - K_3)\, a' - K_2\, b'\, \frac{a'}{a'+K} + K_5\,,
\end{aligned} \tag{1}$$

where a, b, a', b' denote the concentration of the two morphogens A and B in compartment 1 (unprimed) and 2 (primed), respectively. D is the diffusion coefficient for the morphogen B, and K represents the phenomenological Michaelis-Menten constant. We note that the above equations presuppose isothermy, homogeneity in either compartment, and fast relaxation of intermediate products as usual.

The two-compartment structure of the 4-D flow in (1) is capable of generating spontaneous symmetry-breaking phase transitions in the symmetrical homogeneous reaction system. Moreover, the differentiated system flow may further bifurcate from stable morphogenesis to boiling-type turbulence [3]. Numerical evidence and analysis of the spatio-temporal correlation between the two compartments under variation of the control parameter K_5 have been presented elsewhere in detail [4, 5].

Let us now see whether recent experimental investigations on spatio-temporal nonlinear transport phenomena in the avalanche breakdown regime of current-carrying semiconductors [7] can be interpreted in terms of the main behavioral characteristics of the present RT reaction-diffusion model. Our experimental semiconductor system consists of a single homogeneously p-doped germanium crystal, electrically driven into the post-breakdown regime

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due to impurity impact ionization at liquid-helium temperatures. Spatially separated and coupled sample subsystems are realized by means of an appropriate arrangement of ohmic contact probes, dividing the semiconductor into different parts which by themselves show nonlinear dynamical behavior.

Based on the argumentation given by Mitin [8] and Schöll et al. [9], a concrete physical mechanism for the self-organized cooperative transport processes in our semiconductor system can be the following. The mobile charge carriers (holes in p-germanium) are heated up by the applied electric field. When their energy is large enough to exceed the ionization energy of the shallow impurities, impact ionization of carriers bound at the impurities occurs. As soon as the probability of impact ionization prevails over recombination, the number of charge carriers increases like an avalanche (avalanche breakdown). The threshold-like onset of the scattering probability due to optical phonons beyond a characteristic energy level abruptly decreases the mean energy of the ionizing carriers. The emission of optical phonons immediately returns the hot carriers from the "active" high-energy region to the "passive" very-low-energy region ("cooling effect"), where they are arrested by inelastic impurity scattering. (Note that the probability of elastic scattering on acoustical phonons can be neglected in the very-low-energy region [8].) The accumulation of the cooled carriers in the passive low-energy region leads to a decrease of the ionization rate with increasing electric field, until recombination prevails over ionization again, decreasing the number of charge carriers. This completes the cycle of oscillating carrier density and mean carrier energy. Further on, diffusive coupling between spatially separated hot carrier subsystems is effected through energy exchange via energy relaxation to the acoustical phonon system and rapid exchange of acoustical phonons between crystal parts. The long-range phonon propagation benefits from the large lattice heat conductivity of germanium. Experimental evidence of the proposed nonlinear transport behavior demonstrated in a variety of symmetry-breaking phase transitions and chaotic scenarios [7] suggests a reaction scheme for the semiconductor system qualitatively similar to that of the phenomenological RT model indicated in Figure 1.

Specifically, the morphogen concentrations and effective rate constants of the present RT model can be interpreted as follows. The activator variable a represents the number density of moving charge carriers reflected in the electric current, whereas the inhibitor variable b corresponds to the mean energy per carrier. The constants K_1 and K_3 give a linear approximation of the impact ionization and recombination rate of the charge carriers, respectively. The term $K_2\, b/(a+K)$ reveals the cooling of the carriers with increasing electric field as an energy (and carrier) dependent scattering probability of the charge carriers due to optical phonons. The rate constant K_4 describes the heat transfer into the liquid-helium bath, the diffusion coefficient D the energy exchange between different hot carrier subsystems via long-range acoustical phonon propagation. Finally, the control parameter K_5 is referable to the electric power input rate regulated by the applied bias voltage.

To conclude, the dynamical possibilities of our synergetic semiconductor system turn out to be directly projected onto the main behavioral characteristics of the generic RT reaction-diffusion model. In particular, we found an interesting analogy between the corresponding system variables and constants. Nevertheless, the rather complicated nonlinear behavior of the autocatalytic impact ionization and inhibiting optical phonon scattering process of the charge carriers can be only roughly approximated with the simple kinetics of the activator model variable. Clearly, the development of a theoretical model for the detailed understanding of the carrier transport phenomena in an extrinsic semiconductor at low temperatures still represents a challenging task.

The authors gratefully acknowledge stimulating discussions with R. P. Huebener, E. Schöll, V. V. Mitin, and W. Metzler and partial support of the Stiftung Volkswagenwerk.

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Verantwortlich für den Inhalt: A. KLEMM
Satz und Druck: Konrad Triltsch, Würzburg